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Hybrid Beamforming Design based on Low Complexity Alternating Minimizing Solution for Millimeter Wave MIMO Systems

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***Abstract*—Millimeter-wave (mmWave) bands have deservedly taken their place in the development of next-generation network communications to address the explosive growth of mobile network data traffic and high-speed communications requirements. Also, the fully connected hybrid architecture can offer a suitable beamforming gain for overcoming the limitations of mmWave signals in outdoor communications. Consequently, for the feasibility of the hybrid architecture implementation, the designing of the optimal analog/digital beamformers is indispensable for providing good performance with the lowest complexity. Therefore, in this work, we develop a low-complexity alternating minimization design using a complete closed-form least-square method in the iterative design process to derive the optimal analog and digital beamformers by considering the total power constraint at the transmitter and the unit constant modulus constraints. Thanks to the offered global optimal solutions and the fast convergence of the least squares method, numerical results demonstrated that the proposed design attains perfect performance with the lowest computational complexity compared to the state-of-art design algorithms in terms of spectral efficiency (SE) in different scenarios.**

***Index Terms*—Fully connected hybrid architecture, analog and digital beamforming design, mmWave MIMO communications, iterative alternating minimization algorithm, complete closed-form least-square solutions.**

I. INTRODUCTION

The development of wireless communication technologies becomes inevitability and essential to satisfy the need for high-quality communication with fast data throughput, low latency, low jitter, great mobility, and high quality of service (QoS) [1]. In order to deal with these requirements, millimeter wave (mmWave) frequency bands 30-300 GHz are a crucial solution for next-generation wireless communication systems to support multi Gigabit-per-second (Gbps) data rate transmission thanks to their vastly unused spectrum. On the one hand, smaller wavelengths of mmWave signals have high carrier frequencies, the main drawbacks of outdoor mmWave wireless cellular communications are huge attenuations and high penetration losses, which significantly limits the system performance and cell coverage [2]. Fortunately, on the other hand, the small wavelength of mmWave signals allows the modeling of massive multiple input multiple output (MIMO) configurations at transceivers which will provide high spatial processing gains that can theoretically achieve the reasonable link margin [3][4]. However, the main challenges to benefit advantages of MIMO technologies in mmWave communication devices are power consumption, hardware complexity, and costs [5]. To address these challenges, hybrid beamforming architectures have attracted recently a lot of research interests to enable the practical deployment of mmWave MIMO communication devices [6] [7]. In fact, hybrid beamforming architectures reduce the power and complexity of transceiver systems by dividing beamforming processing into digital and analog beamformers which are normally realized using phase shifters in fully connected hybrid architecture [8]. However, the joint design of optimal analog/digital beamformers is a hard problem on each transmission side to achieve an adequate spectral efficiency (SE) results. Moreover, using phase shifters in the implementation of analog beamformers imposes constant modulus constraints which engender a high non-convexity in the design problem where its solution becomes hard. To solve this problem, several works have been proposed to design the analog and digital beamforming. In [9], the authors proposed spatially sparse methods via the Orthogonal Matching Pursuit (OMP) algorithm to design analog and digital beamformers by leveraging the sparse nature of the mmWave channel. Nevertheless, SE results obtained by [9] have attained low performance compared with optimal full-digital beamforming as a benchmark [6]. Instead of the sparse reconstruction-based solutions, other design methods are proposed as a matrix factorization problem to find optimal analog and digital beamformers. In [10], the authors developed alternating minimization (AltMin) algorithms using Manifold Optimization (MO-AltMin) and Phase Extraction (PE-AltMin) approaches to fulfill the design requirement of analog and digital beamformers. Although the improved SE results, MO-AltMin and the PE-AltMin designs require high computational complexity which is prohibitive, especially in mmWave communication where we need large antenna arrays to achieve high-quality communication links. Therefore, we develop in this work a low complexity alternating minimizing design using a complete closed-form least-square method in the iterative design process to derive the optimal analog and digital beamformers by taking into account the total power constraint at the transmitter and unit constant modulus constraints according to the fully connected hybrid architecture. By leveraging global optimal solutions given by the least squares method, the proposed design offers near perfect performance compared to the ideal design. In addition, the reduced number of iterations necessary for converging the alternating minimizing optimization allow us to keep the computational complexity of the proposed design with low-cost whatever the hybrid architecture complexity. Simulation results demonstrate that the proposed design can obtain perfect performance with the lowest complexity compared to the existing design methods in different scenarios.

The remainder of this paper is organized as follows. Section II defines the System and Channel Model of fully connected hybrid architecture for multistream communication. Section III depicts the details of the proposed method which is based on a complete closed-form least-square solution for designing analog/digital precoder/combiner. Comparing simulation results are illustrated in Section IV, and finalized by conclusion in Section V.

II. SYSTEM MODEL

We consider the fully connected mmWave MIMO hybrid architecture shown in Fig.1 as a point-to-point multistream mmWave communication system throughout the paper. The transmitter $(\mathtt{Tx})$ is powered by $N_{\mathtt{tx}}$ element antennas and $N_{RF}^{\mathtt{tx}}$ RF chains for sending N_s multi-stream data in the downlink communication with time-division duplexing protocol (TDD) assumption. The receiver $(\mathtt{Rx})$ has $N_{\mathtt{rx}}$ antennas and $N_{RF}^{\mathtt{rx}}$ RF chains, where the number of multi-stream data must be constrained by $N_s \leq N_{RF}^{\mathtt{tx}} \leq N_{\mathtt{tx}}$ and $N_s \leq N_{RF}^{\mathtt{rx}} \leq N_{\mathtt{rx}}$, respectively. For hybrid beamforming communication in mmWave MIMO systems, the $\mathtt{Tx}$ exploits the digital precoder $\boldsymbol{F_{BB}} \in \mathbb{C}^{N_{RF}^{\mathtt{tx}} \times N_s}$ and analog RF precoder $\boldsymbol{F_{RF}} \in \mathbb{C}^{N_{\mathtt{tx}} \times N_{RF}^{\mathtt{tx}}}$ for providing multi-data transmissions. As $\boldsymbol{F_{RF}}$ is implemented by analog phase shifters, its elements are contrained to satisfy $([\boldsymbol{F_{RF}}]_{:,i}[\boldsymbol{F_{RF}}]_{:,i}^{H})_{\ell,\ell} = N_{\mathtt{tx}}^{-1}$, which are subject to unit constant modulus constraints, where $(\cdot)_{\ell,\ell}$ denotes the ℓ^{th} diagonal element. For detecting the transmitted multi-stream data via $\mathtt{Tx}$ at the receiver side, the $\mathtt{Rx}$ uses an $N_{\mathtt{rx}} \times N_{RF}^{\mathtt{rx}}$ RF combiner $\boldsymbol{W}_{RF}$ followed by an $N_{RF}^{\mathtt{rx}} \times N_s$ baseband combiner $\boldsymbol{W}_{BB}$ to process the transmitted signal $\boldsymbol{x} \in \mathbb{C}^{N_s \times 1}$ through the mmWave MIMO channel. Therefore, the received signal vector $\boldsymbol{y} \in \mathbb{C}^{N_{RF}^{\mathtt{rx}} \times 1}$ can be rewritten as follows:

$$\boldsymbol{y} = \sqrt{\gamma}\boldsymbol{W_{BB}^{H}}\boldsymbol{W_{RF}^{H}}\boldsymbol{H}\boldsymbol{F_{RF}}\boldsymbol{F_{BB}}\boldsymbol{x} + \boldsymbol{W_{BB}^{H}}\boldsymbol{W_{RF}^{H}}\boldsymbol{\eta} \tag{1}$$

where γ indicates the average transmit power, and $\boldsymbol{\eta} \sim \mathcal{CN}(0, \sigma_{\boldsymbol{\eta}}^2\boldsymbol{I})$ is the additive noise vector. Whereas, $\boldsymbol{H} \in \mathbb{C}^{N_{\mathtt{rx}} \times N_{\mathtt{tx}}}$ denotes the mmWave MIMO channel matrix. We adopt the Saleh-Valenzuela geometric narrowband formulation, where the mmWave MIMO channel between $\mathtt{Tx}$ and $\mathtt{Rx}$ can be expressed with N_{cl} scattering clusters and each with N_{ray} paths as follows [9][6] :

$$\boldsymbol{H} = \rho \sum_{i=1}^{N_{cl}} \sum_{l=1}^{N_{ray}} \alpha_{il}\boldsymbol{a}_{\mathtt{rx}}(\phi_{il}^{\mathtt{rx}}, \theta_{il}^{\mathtt{rx}})\boldsymbol{a}_{\mathtt{tx}}(\phi_{il}^{\mathtt{tx}}, \theta_{il}^{\mathtt{tx}})^{H} \tag{2}$$

where α_{il} indicates complex gain of l^{th} path channel in the i^{th} clusters by assuming that channel gains follow the complex Gaussian distribution, i.e., $\alpha_{il} \sim \mathcal{CN}(0, \sigma_{\alpha,i}^2)$, where $\sum_{i=1}^{N_{cl}} \sigma_{\alpha,i}^2 = \rho$, whereas $\rho = \sqrt{N_{\mathtt{tx}}N_{\mathtt{rx}}/N_{cl}N_{ray}}$ represents the factor for normalizing the channel, such as $\mathbb{E}\left[\|\boldsymbol{H}[k]\|_F^2\right] = N_{\mathtt{tx}}N_{\mathtt{rx}}$. Concerning $\boldsymbol{a}_{\mathtt{rx}}(\phi_{il}^{\mathtt{rx}}, \theta_{il}^{\mathtt{rx}})$ and $\boldsymbol{a}_{\mathtt{tx}}(\phi_{il}^{\mathtt{tx}}, \theta_{il}^{\mathtt{tx}})$ are esteemed in this channel model as receive and transmit steering vectors for elevation (azimuth) angle of $\theta_{il}^{\mathtt{tx}}(\phi_{il}^{\mathtt{tx}})$ and $\theta_{il}^{\mathtt{rx}}(\phi_{il}^{\mathtt{rx}})$ respectively. And by suggestion the uniform planar array (UPA) configuration is used in our study. $\boldsymbol{a}_{\mathtt{tx}}(\phi_{il}^{\mathtt{tx}}, \theta_{il}^{\mathtt{tx}})$ and $\boldsymbol{a}_{\mathtt{rx}}(\phi_{il}^{\mathtt{rx}}, \theta_{il}^{\mathtt{rx}})$ are rewritten as

$$\begin{aligned}
\boldsymbol{a}_{\mathtt{tx}}(\phi_{il}^{\mathtt{tx}}, \theta_{il}^{\mathtt{tx}}) &= \tfrac{1}{N_{\mathtt{tx}}}[1, \ldots, e^{j\frac{2\pi}{\lambda}d(p\sin(\phi_{il})\sin(\theta_{il})+q\cos(\theta_{il}))}, \\
&\ldots, e^{j\frac{2\pi}{\lambda}d((\sqrt{N_{\mathtt{tx}}}-1)\sin(\phi_{il})\sin(\theta_{il})+(\sqrt{N_{\mathtt{tx}}}-1)\cos(\theta_{il}))}] \\
\boldsymbol{a}_{\mathtt{rx}}(\phi_{il}^{\mathtt{rx}}, \theta_{il}^{\mathtt{rx}}) &= \tfrac{1}{N_{\mathtt{rx}}}[1, \ldots, e^{j\frac{2\pi}{\lambda}d(p\sin(\phi_{il})\sin(\theta_{il})+q\cos(\theta_{il}))}, \\
&\ldots, e^{j\frac{2\pi}{\lambda}d((\sqrt{N_{\mathtt{rx}}}-1)\sin(\phi_{il})\sin(\theta_{il})+(\sqrt{N_{\mathtt{rx}}}-1)\cos(\theta_{il}))}]
\end{aligned} \tag{3}$$

where λ is the mmWave signal wavelength. d indicates the separation distance of array elements. p and q represent the indices of the antenna elements in the 2D plane.

III. PROPOSED PRECODING AND COMBINING DESIGNS IN A HYBRID ANALOG AND DIGITAL ARCHITECTURE

For hybrid beamforming systems, the SE can be formulated by assuming that transmitted symbols follow a Gaussian distribution as

$$\begin{aligned}
\boldsymbol{R} = \log_2\Bigg(\Bigg|\boldsymbol{I}_{N_s} + \frac{\gamma}{N_s}\boldsymbol{R}_n^{-1}\boldsymbol{W_{BB}^{H}}\boldsymbol{W_{RF}^{H}}\boldsymbol{H}\boldsymbol{F_{RF}}\boldsymbol{F_{BB}} \\
\times \boldsymbol{F_{BB}^{H}}\boldsymbol{F_{RF}^{H}}\boldsymbol{H}^{H}\boldsymbol{W_{RF}}\boldsymbol{W_{BB}}\Bigg|\Bigg)
\end{aligned} \tag{4}$$

where $\boldsymbol{R}_n = \sigma_n^2\boldsymbol{W_{BB}^{H}}\boldsymbol{W_{RF}^{H}}\boldsymbol{W_{RF}}\boldsymbol{W_{BB}}$ is the noise covariance matrix after combining. Therefore, the original design problem of optimal hybrid beamformers $(\boldsymbol{F_{BB}^{opt}}, \boldsymbol{F_{RF}^{opt}}, \boldsymbol{W_{BB}^{opt}}, \boldsymbol{W_{RF}^{opt}})$ can be modeled by formulating a joint optimization problem to maximize the SE as

$$\begin{aligned}
\{\boldsymbol{F_{BB}^{opt}}, \boldsymbol{F_{RF}^{opt}}, \boldsymbol{W_{BB}^{opt}}, \boldsymbol{W_{RF}^{opt}}\} &= \underset{\boldsymbol{F_{BB}}, \boldsymbol{F_{RF}}, \boldsymbol{W_{BB}}, \boldsymbol{W_{RF}}}{\arg\max} \boldsymbol{R} \\
\text{subjet to} &\begin{cases} \|\boldsymbol{F_{RF}}\boldsymbol{F_{BB}}\|_F^2 = N_s \\ |\boldsymbol{F_{RF}}(i,j)| = \sqrt{1/N_{\mathtt{tx}}} \\ |\boldsymbol{W_{RF}}(i,j)| = \sqrt{1/N_{\mathtt{rx}}} \end{cases}
\end{aligned} \tag{5}$$

However, maximizing (5) is a nonconvex problem, and finding directly the global optima is intractable due to the four

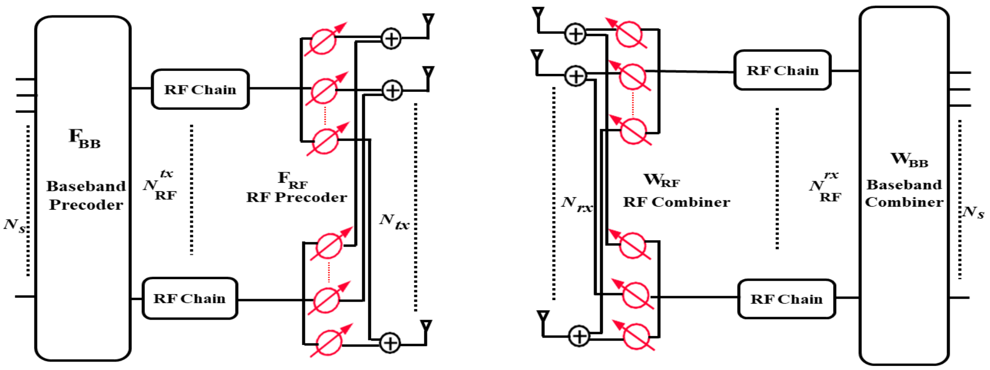

Fig. 1. Fully-connected mmWave MIMO hybrid architecture.

beamforming matrices and the non-convexity of constraints. To facilitate the solutions of joint hybrid beamformer designs, the joint optimization problem in (5) can be transformed into two separate optimization design problems with disassociated constraints at each transceiver side as demonstrated in [9]. The optimal hybrid beamformers that maximize SE (5) are equivalent to the Frobenius-norm-minimization problem between the optimal unconstrained digital beamformers which is derived from the channel's singular value decomposition (SVD), (i.e. $\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}$), and analog/digital beamformers that will be designed. At `Tx`, the problem formulation for designing $\boldsymbol{F_{BB}}$ and $\boldsymbol{F_{RF}}$ can be written as

$$\{\boldsymbol{F}_{\boldsymbol{BB}}^{opt}, \boldsymbol{F}_{\boldsymbol{RF}}^{opt}\} = \underset{\boldsymbol{F_{BB}},\boldsymbol{F_{RF}}}{\arg\min} \|\boldsymbol{F}_{opt} - \boldsymbol{F_{RF}}\boldsymbol{F_{BB}}\|_F \qquad (6)$$

$$\text{subjet to} \begin{cases} \|\boldsymbol{F_{RF}}\boldsymbol{F_{BB}}\|_F^2 = N_s \\ |\boldsymbol{F_{RF}}(i,j)| = \sqrt{1/N_{\mathtt{tx}}} \end{cases}$$

where $\boldsymbol{F}_{opt} \in \mathbb{C}^{N_{\mathtt{tx}} \times N_s}$ represents the optimal unconstrained digital precoders which consists of the first N_s columns of matrix $\boldsymbol{V}$. While at the `Rx` side, the combining problem design of $\boldsymbol{W_{BB}}$ and $\boldsymbol{W_{RF}}$ can also be formulated as

$$\{\boldsymbol{W}_{\boldsymbol{BB}}^{opt}, \boldsymbol{W}_{\boldsymbol{RF}}^{opt}\} = \underset{\boldsymbol{W_{BB}},\boldsymbol{W_{RF}}}{\arg\min} \|\boldsymbol{W}_{opt} - \boldsymbol{W_{RF}}\boldsymbol{W_{BB}}\|_F \qquad (7)$$

$$\text{subjet to} \, |\boldsymbol{W_{RF}}(i,j)| = \sqrt{1/N_{\mathtt{rx}}}$$

where $\boldsymbol{W}_{opt} \in \mathbb{C}^{N_{\mathtt{rx}} \times N_s}$ is the optimal unconstrained digital combiners corresponding to the first N_s columns of matrix $\boldsymbol{U}$. To design analog and digital beamformers, we exploit a complete closed-form least-square solution as mentioned above for developing algorithms based on an iterative alternating minimization optimization. By neglecting constraints on the problem design and facilate the fixing analog precoder $\boldsymbol{F_{RF}}$ in the beginning [1]. Wherefore, the optimization problem in (6) will become a simple linear least squares problem to find and update the digital precoders with the unique solution as $\boldsymbol{F_{BB}} = (\boldsymbol{F}_{\boldsymbol{RF}}^H \boldsymbol{F_{RF}})^{-1} \boldsymbol{F}_{\boldsymbol{RF}}^H \boldsymbol{F}_{opt}$. After, we fix the designed digital precoder to optimize the analog precoder in the same manner where the obtained solution can be expressed as $\boldsymbol{F_{RF}} = \boldsymbol{F}_{opt}\boldsymbol{F}_{\boldsymbol{BB}}^H(\boldsymbol{F_{BB}}\boldsymbol{F}_{\boldsymbol{BB}}^H)^{-1}$. To respect unit constant modulus constraints, we normalize each analog precoder vector $\boldsymbol{F_{RF}}(:,j)$ for all $N_{RF}^{\mathtt{tx}}$ analog beamforming vectors by the factor of $\frac{\sqrt{N_{\mathtt{tx}}}}{\|\boldsymbol{F_{RF}}(:,j)\|_2}$. After the convergence of the iterative alternating minimization optimization, all we need now is to scale the obtained $\boldsymbol{F_{BB}}$ by a factor of $\frac{\sqrt{N_s}}{\|\boldsymbol{F_{BB}}\boldsymbol{F_{RF}}\|_F}$ to satisfy the power constraint. All steps used to design analog and digital precoders are summarized by Algorithm 1 which can be used in the the design of analog and digital combiners.

Algorithm 1 LS-AltMin-Based Analog and Digital Precoding Design

```
Input:
1) $\boldsymbol{H}$; $[\boldsymbol{U},\boldsymbol{\Sigma},\boldsymbol{V}] = \mathrm{svd}(\boldsymbol{H}), N_{\mathtt{tx}}, N_s,\ N_{RF}^{\mathtt{tx}}$
2) $\boldsymbol{F}_{opt} = \boldsymbol{V}(:,1:N_s)$;
3) Generate $\boldsymbol{F}_{RF}^{(0)}$ using uniform random phase
4) Set $k = 1$                    %% iteration index
Repeat
5) Fix $\boldsymbol{F}_{RF}^{(k-1)}$
6) Compute $\boldsymbol{F}_{BB}^{(k)} = (\boldsymbol{F}_{\boldsymbol{RF}}^{(k-1)^H} \boldsymbol{F_{RF}})^{(k-1)} \boldsymbol{F}_{\boldsymbol{RF}}^{(k-1)^H} \boldsymbol{F}_{opt}$
7) Fix $\boldsymbol{F}_{BB}^{(k)}$
8) Compute$\boldsymbol{F}_{\boldsymbol{RF}}^{(k)} = \boldsymbol{F}_{opt}\boldsymbol{F}_{\boldsymbol{BB}}^{(k)^H}(\boldsymbol{F}_{\boldsymbol{BB}}^{(k)}\boldsymbol{F}_{\boldsymbol{BB}}^{(k)^H})^{-1}$
   for $i \le N_{RF}^{\mathtt{tx}}$ do
       $\boldsymbol{F}_{\boldsymbol{RF}}^{(k)}(:,j) = \sqrt{N_{\mathtt{tx}}}\frac{\boldsymbol{F}_{\boldsymbol{RF}}^{(k)}(:,j)}{\left\|\boldsymbol{F}_{\boldsymbol{RF}}^{(k)}(:,j)\right\|_2}$
    end for
9) $k \leftarrow k + 1$
until a stopping criterion triggers
10) Normalization step of digital precoder as:
   $\boldsymbol{F_{BB}} = \sqrt{N_s}\frac{\boldsymbol{F_{BB}}}{\left\|\boldsymbol{F}_{RF}\hat{\boldsymbol{F}}_{\boldsymbol{BB}}\right\|_F}$
```

IV. SIMULATION RESULTS

In this section, we will evaluate the robustness of the proposed LS-AltMin algorithm for designing analog and digital beamformers and comparing its performance with the obtained results by the proposed method in [9], [10], and the hybrid design based-OMP algorithm in terms of SE for different scenarios. The simulation parameters are chosen according

---

[1] As the realization structure of the analog architecture part is based on phase shifters. We can generate the analog precoders (or combiners) as $[\boldsymbol{F_{RF}}]_{(i,j)} = \frac{1}{\sqrt{N_{\mathtt{tx}}}} e^{\varphi_{i,j}}$ where $\varphi_{i,j} \in [0, 2\pi]$ is the phase of i^{th} analog precoder column, such that, $\varphi_{i,j}$ can be gererated with uniform random distribution

to the fully connected hybrid architecture as illustrated in Fig.1 that is dedicated for multi-data streams communications with $N_s = 4$, where the transceiver has the size antenna arrays of $N_{\mathtt{tx}} \times N_{\mathtt{rx}} = 64 \times 16$, and the same number of RF chains at each side as $N_{RF}^{\mathtt{tx}} = N_{RF}^{\mathtt{rx}} = 5$. For the mmWave MIMO channel, the number of scattering clusters and paths are $N_{cl} = 5$ and $N_{ray} = 10$, respectively. Where the average power of each cluster $\sigma_{\alpha,i}^2$ is equal to 1. With the suggestion of UPA configuration, the AoDs and AoAs are assumed to follow the Laplacian distribution with uniformly distributed mean angles with angular spread of $10°$ for both the `Tx` and `Rx`. Fig.2 illustrates the SE performance versus SNR for the proposed algorithm and other hybrid design methods developed in [9], [10], hybrid design based-OMP algorithm, and analog-only scheme. As shown in Fig. 2, the proposed analog/digital beamforming design outperforms all other design techniques by performing near-optimal to optimal full-digital precoding/combining over the whole SNR range.

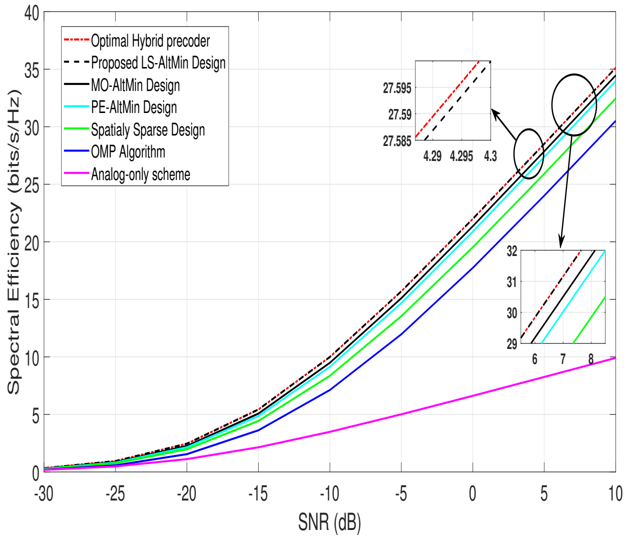

Fig. 2. Comparison of SE results obtained by different analog and digital beamforming design algorithms.

Fig.3 provides the SE results versus the number of RF chains where the SNR and N_s are set to $5dB$ and 4 data streams, respectively, for all designed analog/digital beamforming methods. From this figure, the proposed design still achieves near-optimal results compared to full-digital precoding/combining performance with the same small gap for all different RF chain values, thanks to global optimal beamformer designs offered by the least squares solutions. As known, increasing the RF chain improves SE results to approach the performance of fully digital architecture. However, the other design algorithms have less performance despite the increasing the RF chain numbers. Since computational complexity must be taken into account in the design process. We assess the complexity of the proposed design by providing a comparison with other design schemes' complexity in terms of the runtime needed for the precoding/combining design algorithm to execute. The complexity evaluation is governed by the computation of the analog and digital beamformers to gather. Fig.4 depict the runtime of the proposed LS-AltMin design, MO-AltMin, PE-AltMin, spatially sparse design, and design-based OMP for a variety of RF chain numbers at $SNR = 5dB$. As illustrated in Fig.4, we can confirm that the proposed design has significantly lower complexity than the other beamforming designs because our designs are based on simple solutions and consume small iterative numbers to converge the alternating minimizing optimization. While the MO-AltMin algorithm has high complexity which is the main limitation for designing the analog/digital precoders/combiners.

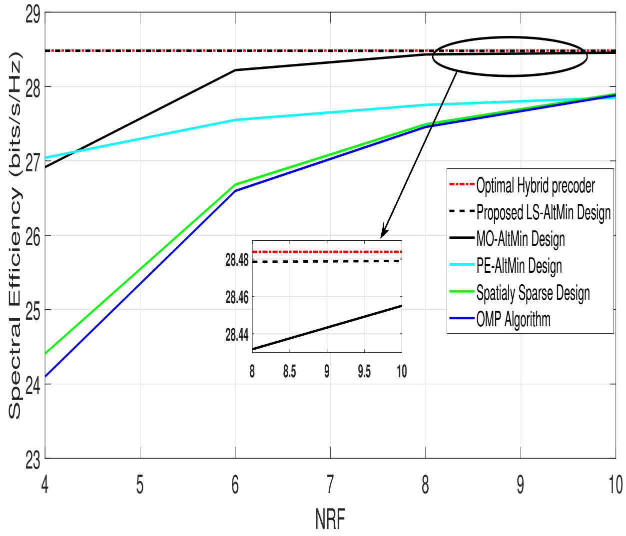

Fig. 3. Obtained SE results of different analog and digital beamforming design algorithms versus RF chain numbers.

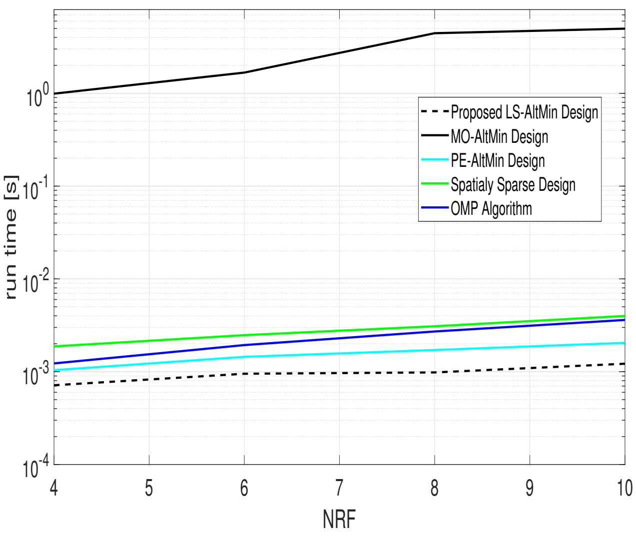

Fig. 4. Runtime comparison of different precoding/combining methods versus RF chain numbers.

For examining transmit antennas' effects on SE performance in a fully connected hybrid architecture, we represent in Fig.5 SE results as a function of transmit antennas with $N_s = 4$ and

RF chain numbers equal to 5 at each side. The performance of all the design algorithms are increasing throughout the antenna numbers where the proposed design maintains the best results with similar behavior and a tiny gap compared to optimal full-digital precoding/combining.

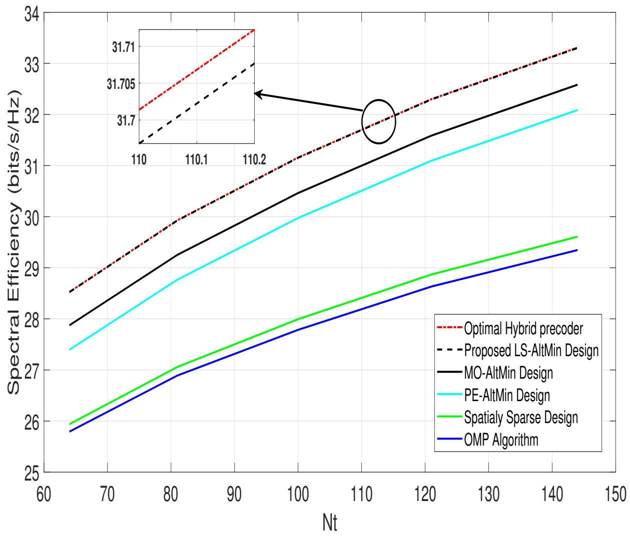

Fig. 5. Obtained SE results of different analog and digital beamforming design algorithms versus the number of transmit antenna.

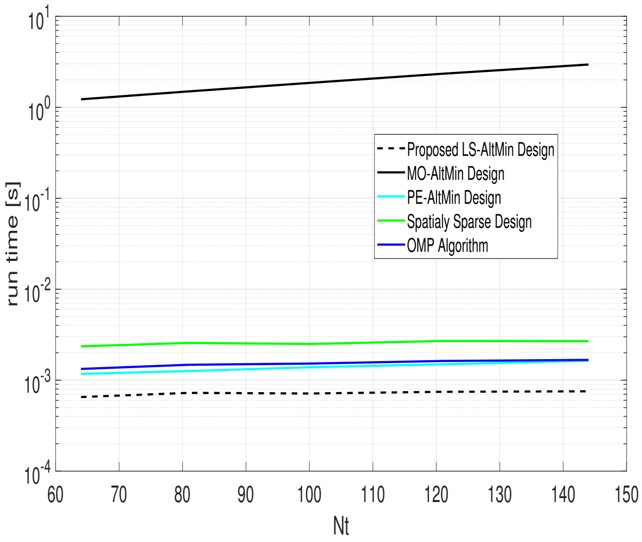

Fig. 6. Runtime comparison of different precoding/combining methods versus the number of transmit antenna.

For the computational complexity performance of SE as a function of transmit antennas, the proposed design has lower complexity because it has linear solutions whatever the increase of transmit antenna numbers as depicted in Fig.6.

V. CONCLUSION

In this paper, we proposed an iterative alternating minimizing algorithm based on a complete closed-form least-square solution for designing analog/digital precoder/combiner, where the total power constraint at the transmitter and unit constant modulus constraints are satisfied algorithmically for a fully connected hybrid architecture. Thanks to globally optimal solutions offered by the least squares method, the proposed design guarantees near-optimal performance to the ideal design. Due to the small iterative numbers consumed for converging the alternating minimizing optimization, the computational complexity is reduced which makes our design more suitable when we need large antenna arrays in mmWave MIMO devices. Numerical results demonstrated that the proposed design obtains good performance with the lowest computational complexity compared to the state-of-art design algorithms in terms of SE.

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